Benchmarking Asset Correlations

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Abstract

Among the most crucial input parameters for credit portfolio risk models are the co-movements of default risks. Due to limited empirical evidence about the magnitude of correlations the New Basel Capital Accord sets standard requirements for calculating regulatory capital requirements, e.g. in the Consultative Document as of April 2003 asset correlations for sovereigns, banks and corporates between 12% and 24% depending on default probabilities are assumed. The present contribution shows how correlations can be estimated within the framework of the Basel II model. Using default data from the G7 countries it is shown that asset correlations are much lower than broadly assumed. This may have valuable consequences for backtesting of PD forecasts carried out by banks and supervisors. Furthermore it is shown how current credit risk models can be parameterized with our estimates. We find that the differences between the model outcomes may become even more negligible than found up to now, thus reducing model risk.

1 Introduction

While approaches for estimating default probabilities have been considerably improved during the last years the analysis of co-movements between borrowers is still in its infancy. Therefore internal credit risk models with a bank's own estimates for correlations are indefinitely not expected to be employed for the calculations of capital requirements. Instead the New Basel Capital Accord assumes standard specifications for correlations which can be interpreted as conservative guidelines for the unknown magnitudes of these parameters.

The main direction of modeling and estimating correlations follows the seminal work due to Merton (1974) who explains a default event by the value of a firm crossing a default threshold. The presumed process for the asset value of a single firm can be extended to a multidimensional process where the correlations between the log-returns on the firms' assets (asset correlations thereafter) are the drivers for the co-movements, see Zhou (2001). Default correlations can be derived by the threshold model. A direct estimation of these parameters however, is doomed by the unobservability of asset values. This problem is bypassed by CreditMetrics who approximate asset returns by equity returns. Notwithstanding the quality of this substitution, marketable equity has to be observed nevertheless – a circumstance which is not fulfilled for most small and medium sized borrowers.

Being aware of this problem, recently Nagpal/Bahar (2001) presented estimates for default correlation between US corporate obligors using a nonparametric approach which was suggested by Lucas (1995). The approach has the main advantage that asset values do not have to be observed, instead a time-series of defaults is sufficient. The present contribution introduces an alternative, parametric approach for the estimation of default correlations which may exhibit several advantages over the one proposed by Lucas (1995). Firstly, the approach can make use of time-varying default probabilities. While the methodology due to Lucas (1995) assumes that default probabilities are constant (see Lucas, 1995, p.82) we explicitly allow for variations over the business cycles due to background factors. As a consequence, default correlations are also modeled in dependence on the state of the economy. The major effect is that much of the fluctuations through time are attributed to the respective point of the cycle rather than to unobservable random factors, thus reducing uncertainty in forecasts for default probabilities and loss distributions. Secondly, the model is a variant of the Basel II factor model which is used for calibrating risk weights and which was introduced by CreditMetrics in the spirit of the Merton model. Therefore, asset or default correlations can be interpreted straightforward in the context of Basel II and the estimates can be compared to the guidelines from the Basel Committee. Furthermore, the parameter estimates can be easily incorporated as input parameters into popular credit risk models, such as CreditMetrics or CreditRisk+. Thirdly, since the approach is parametric, one can easily compute confidence intervals for the parameters and conduct significance tests. This may be important when estimation risk is considered for the forecasts. We apply our methodology to a large database of industry-specific defaults in the G7 countries and provide benchmarks for international asset and default correlations which can be employed by banks in credit risk models or used by supervisors for backtesting of default probability forecast.

The rest of the paper proceeds as follows. Section 2 provides an overview of the models and the estimation methodologies. Section 3 describes the data and the main estimation results for the G7 countries. Section 4 shows how the results can be used in practice by banks and supervisors.

2 Modeling and Estimating Correlations

2.1 Setup of the models

We use a variant of the factor model from Credit Metrics which is employed in the framework of Basel II for calibrating risk weights. The normalized return R_{it} on a firm *i*'s assets at time *t* is assumed to follow a one factor model of the form

$$R_{it} = b\, F_t + \sqrt{1-b^2}\, U_{it} \qquad (*\ 1).$$

where

$$F_t \sim N(0,1), \quad U_{it} \sim N(0,1)$$

(i=1,…, N_t , t=1,...,T) are normally distributed with mean zero and standard deviation one. Idiosyncratic shocks U_{it} are assumed to be independent from the systematic factor F_t and independent for different borrowers. All random variables are serially independent. The exposure to the common factor is denoted by b. Under these assumptions the correlation between the normalized asset returns of any two borrowers is b^2. For example in the consultative paper of January 2001 this correlation is set to 0.2. In the document as of April 2003 the asset correlation is a function of PD, and of PD and firm size respectively. The asset correlations can be transformed into default correlations as it is shown in Koyluoglu/Hickman (1998).

A borrower defaults at time t if his return falls short of some threshold β_0, i.e.

$$R_{it} < \beta_0 \quad \Leftrightarrow \quad Y_{it} = 1 \qquad (* \ 2).$$

(i=1,…, N_t , t=1,...,T), where Y_{it} is an indicator variable with

$$Y_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults at time } t \\ 0 & \text{else} \end{cases}$$

The probability of default at time t for borrower i is then

$$\lambda = P(Y_{it} = 1) = P(R_{it} < \beta_0) = P\left(b\, F_t + \sqrt{1-b^2}\, U_{it} < \beta_0\right) = \Phi(\beta_0) \qquad (* \ 3).$$

where $\Phi(.)$ denotes the cumulative standard normal distribution function. This probability is actually a conditional probability, given the borrower has survived until time t. We skip the condition $Y_{it-1} = 0$ for convenience. Conditional on a realization f_t of the common random factor at time t the (conditional) default probability becomes

$$\lambda(f_t) = P\left(U_{it} < \frac{\beta_0 - b\, f_t}{\sqrt{1-b^2}} \right) = \Phi\left(\frac{\beta_0 - b\, f_t}{\sqrt{1-b^2}} \right) \qquad (*\ 4).$$

As described in Finger (1998), the realization f_t of the factor can be interpreted as a kind of "macroeconomic condition". In "good years" - that is, a positive factor realization - the conditional default probabilities decrease whereas they increase in "bad years". The unconditional default probability λ is the expectation of the conditional probability regarding the random factor, i.e. $\int_{-\infty}^{+\infty} \lambda(f_t)\, \varphi(f_t)\, df_t$ where $\varphi(.)$ denotes the density function of the standard normal distribution. Conditional on the realization of the random factor defaults are independent between borrowers.

In (* 3) the unconditional PD is assumed to be constant over time. Hereby it is assumed that all fluctuations of default risks are due to the random factor, that is, higher conditional PD's in "bad years" are solely the result of a "bad" factor realization. By doing so we attribute all co-movements to the asset correlation.

In contrast to completely attributing fluctuations of default rates to the random factor it can be taken into account that next year's default rates are associated with this year's default rates if this year's default rates can be seen as proxies for the current state of the economy. For example, Duffie/Singleton (1999), or Duffee (1999) suggest an AR(1)-process for the default intensities. Then year t's default probability becomes a function of the default rate DR_{t-1} of year $t-1$ which is calculated as the number of defaults in $t-1$ divided by the number of companies in $t-1$, i.e.

$$\begin{aligned} \lambda_t &= P\left(Y_{it} = 1 \middle| DR_{t-1}\right) = P\left(R_{it} < \beta_0 + \beta_1 DR_{t-1}\right) \\ &= P\left(b\, F_t + \sqrt{1-b^2}\; U_{it} < \beta_0 + \beta_1 DR_{t-1} \right) = \Phi\left(\beta_0 + \beta_1 DR_{t-1}\right) \end{aligned} \qquad (*\ 5)$$

where β_1 is the coefficient for the lagged default rate. As a matter of fact, the lagged default rate represents a proxy for the respective situation at the point of the business cycle and is not necessarily responsible for the default probabilities themselves.

Conditional on a realization f_t of the common random factor at time *t* the default probability is

$$\lambda_t(f_t) = P\left(U_{it} < \frac{\beta_0 + \beta_1 DR_{t-1} - b\, f_t}{\sqrt{1-b^2}} \right) = \Phi\left(\frac{\beta_0 + \beta_1 DR_{t-1} - b\, f_t}{\sqrt{1-b^2}} \right) \qquad (*\ 6).$$

Note that model (* 5) addresses an important shortcoming of the model with constant default probability. The simple model assumes an unobservable i.i.d. random factor which does not have a history. In reality however, a state of an economy naturally depends on its former state. As such, an i.i.d. random factor may be inadequate and all fluctuations are captured by the asset correlation. For this reason, model (* 5) includes the lagged default rate as an observable proxy for the state of the economy and argues that only the unobservable "residual" of the fluctuation could be a kind of random "white noise", the influence of which, or the remaining asset correlation respectively, should be reduced.

2.2 Estimation Approach

Suppose for a given segment (for example an industry sector) we have observed a time series of defaults D_t, and numbers N_t of borrowers (*t*=1,...,*T*). For given realization of the random factor in *t* the defaults are independent, that is, within a homogenous segment the number of defaults is conditional binomial with N_t and conditional default probability $\lambda(f_t)$. To get the unconditional distribution one has to integrate over the random factor. Since the factor is independently and identically distributed over time, the marginal log-likelihood function for the observed time series $(D_1,...,D_T)$ and $(N_1,...,N_T)$ depends only on the parameters β_0 and b and is

$$l(\beta_0, b) = \sum_{t=1}^{T} \ln \left\{ \int_{-\infty}^{\infty} \binom{N_t}{D_t} \Phi\left(\frac{\beta_0 - b\, f_t}{\sqrt{1-b^2}}\right)^{D_t} \cdot \left[1 - \Phi\left(\frac{\beta_0 - b\, f_t}{\sqrt{1-b^2}}\right)\right]^{(N_t - D_t)} \varphi(f_t)\, df_t \right\}$$

where $\Phi\left(\frac{\beta_0 - b\, f_t}{\sqrt{1-b^2}}\right)$ is the conditional default probability given by (* 4). If a covariate such as the lagged default rate is included, the log-likelihood additionally depends on the parameter β_1 and becomes

$$l(\beta_0, \beta_1, b)$$
$$= \sum_{t=1}^{T} \ln \left\{ \int_{-\infty}^{\infty} \binom{N_t}{D_t} \Phi\left(\frac{\beta_0 + \beta_1 DR_{t-1} - b\, f_t}{\sqrt{1-b^2}}\right)^{D_t} \cdot \left[1 - \Phi\left(\frac{\beta_0 + \beta_1 DR_{t-1} - b\, f_t}{\sqrt{1-b^2}}\right)\right]^{(N_t - D_t)} \varphi(f_t)\, df_t \right\}$$

where $\Phi\left(\frac{\beta_0 + \beta_1 DR_{t-1} - b\, f_t}{\sqrt{1-b^2}}\right)$ denotes the conditional default probability given by (* 6).

As an extension of common logit or probit models an important part of the method is the integral over the random effect. The integral approximation can for example be conducted by the adaptive Gaussian quadrature as it is described in Pinheiro/Bates (1995). Usually this log-likelihood function is numerically optimized with respect to the unknown parameters for which several algorithms, such as the Newton-Raphson method, exist and are implemented in statistical software packages. Under usual regulatory conditions the resulting estimators $\hat{\beta}_0$ and $\hat{b}$, or $\hat{\beta}_1$ respectively, asymptotically exist, are consistent and converge against normality, i.e.

$$\begin{pmatrix} \hat{\beta}_0 \\ \hat{\beta}_1 \\ \hat{b} \end{pmatrix} \overset{a}{\sim} N\left[\begin{pmatrix} \beta_0 \\ \beta_1 \\ b \end{pmatrix}, \boldsymbol{\Sigma}\right]$$

where Σ denotes the covariance matrix of the estimators. The (asymptotic) variance of the asset correlation $\hat{\rho}$ can be obtained by the Delta-method, which is described in Billingsley (1979). Thus common statistical tests for significance can be conducted. More details may be requested from the authors.

3 Correlations in the G7 Countries

For the analysis we use the data which were already used by the study due to Boegelein et al. (2002). These authors also provide a detailed description of the data. Total numbers of enterprises and bankruptcies of the G7 countries were divided into five industry sectors by the International Standard Industrial Classification (ISIC Revision 3). These are Agriculture, Commerce, Construction, Manufacturing, and Services respectively. The encoding can be found in Table 1. While Boegelein et al. (2002) focus on co-movements *between* segments we look at asset correlations of individual corporates *within* each segment with a straightforward interpretation with respect to Basel II.

The longest time series are from Germany and Great Britain with about 20 years of default data. The minimum numbers are from Japan and Canada with 10 years from 1990 to 1999 with exception of segment 1 from Japan with only 3 years. Thus this segment is excluded from our calculations. For segments 2 and 3 in Italy we could not collect any data, therefore these segments are also excluded. Exhibit 1 shows the historical default rates exemplary for the construction segment in Germany, the USA and Great Britain.

---Insert Table 1 about here---

---Insert Exhibit 1 about here---

For the available segments we firstly compare the estimates due to the methodology from Nagpal/Bahar (2001) with our model (* 4). That is, using the entire horizon of observations for each segment we assume constant default probabilities over time and attribute all fluctuations of default rates to the random factor. The realizations of the random factor are treated as occurrences of “good years” or “bad years” respectively. We refer to these approaches as “static models” thereafter. By this we assume a somewhat conservative view. If we additionally incorporate a rating which considers at least in part the state of the economy then fluctuations of defaults within the grades should be more smoothed. Thus, the resulting asset or default correlations can be interpreted as “upper benchmarks” of the average correlation within the countries and segments.

First of all, Table 2 contains the estimation results for the constant and the exposure to the random effect. All estimates are statistically significant at the 1% level except in two segments where they are significant at the 10% level (marked by an asterix). The asset correlation can be calculated as the square of the estimated exposure. Furthermore, we employ the methodology due to Nagpal/Bahar (2001) and estimate default probabilities and default correlations. These default correlations are then converted into asset correlations as shown for example in Koyluoglu/Hickman (1998). Exhibit 2 shows a comparison of the resulting asset correlations for both approaches.

---Insert Table 2 about here---

---Insert Exhibit 2 about here---

For each country and industry sector the left hand bars illustrate the estimates due to the non-parametric static PD approach, the bars in the middle represent the estimates due to the parametric static PD approach (model (* 4)). Firstly, correlations are generally very low and except for sectors 1 in USA and Italy (Agriculture), and sectors 4 and 5 in Canada the estimates from the two models are not exceedingly different. The highest asset correlation with the non-parametric approach is found in the Construction sector of Germany with about 2.1%. Using the parametric approach it can be found in segment 1 in the USA with approximately 2.3%, followed by segments 4 and 3 in Japan and segment 3 in Germany with 1.5%. In Canada and France all correlations are lower than 1%. In Great Britain they are on average only little higher than 1%.

In the next step we estimate model (* 6). Table 3 shows that in most segments the lagged default rate is statistically significant with a positive sign indicating that the actual default rate is partly a good proxy for the state of the economy. As an example Exhibit 3 shows for the construction sector in Germany the actual default rates until year 2001 and the forecasts of the model until year 2002. As easily can be seen, one is better off in forecasting the default probability for year 2002 if one takes the current state of the economy in year 2001 into account instead of taking a simple historical average as forecast.

---Insert Exhibit 3 about here---

Furthermore, the bars on the right hand side in Exhibit 2 summarize the asset correlation of this dynamic model for each country and sector. Compared to the estimates of the static models it is obvious that all asset correlations can be strongly reduced by attributing part of the comovements to the state of the business cycle. The highest remaining asset correlation is still in segment 1 in the United States, but it is reduced to 1.6%. The next highest correlation can be found in Great Britain's segment 4. It reduces from 1.3% to 0.5%. In all countries the average correlation is much lower than with either static model.

---Insert Table 3 about here---

In addition, asset correlations between borrowers from different industry sectors and/or different countries can be estimated. For this, we use the approach suggested in Rösch/Scheule (2003) using US retail portfolio data. Model (* 1) is defined for each asset return $R_{it}^{(l)}$ within segment l (l=1,…,L) (industry/country)

$$R_{it}^{(l)} = b^{(l)} F_t^{(l)} + \sqrt{1 - b^{(l)2}}\, U_{it}^{(l)} \qquad (*\ 7).$$

where

$$F_t^{(l)} \sim N(0,1), \quad U_{it}^{(l)} \sim N(0,1)$$

(i=1,…, N_t , t=1,…,T). The correlation between two asset returns is then

$$Corr\left(R_{it}^{(l)}, R_{jt}^{(s)}\right) = \begin{cases} b^{(l)2} & l = s, i \neq j \\ b^{(l)}\, b^{(s)}\, \rho_{ls} & l \neq s, i \neq j \end{cases} \qquad (*\ 8).$$

where

$$\rho_{ls} \equiv Corr\left(F_t^{(l)}, F_t^{(s)}\right) \qquad (*\ 9).$$

denotes the correlation between the random factors of two different sectors/countries. These factor correlations can be provided by firstly estimating the time series of the factor realizations from the sector specific models and then calculating the correlation coefficient between these time series.

Due to space limitations we only report the asset correlations between the countries for each model, that is, we estimate model (* 7) for each country as a whole, calculate the correlations (* 9) between the factors and finally determine the asset correlations (* 8) between each two borrowers. Exhibits 4 and 5 show the results for the static and for the dynamic model where the diagonals contain the within-country asset correlations. Other results regarding correlations between industry sectors may be requested from the authors.

---Insert Exhibits 4 and 5 about here---

As can be seen from Exhibit 4 correlations in the static model are comparatively high between Canada, USA and Great Britain, and Germany, Italy and Japan. They are remarkably negative between Canada and Germany, and Canada and Japan. If on the other hand the dynamic model is used (Exhibit 5), not only the asset correlations within the countries but also correlations between borrowers from different countries are strongly diminished (absolutely).

4 Implications and Uses

The values of the asset correlations which are assumed in the New Basel Capital Accord are justifiable under a conservative view since empirical evidence on correlations is rather scarce and capital buffers should not be stressed even under a wide potential range of parameter assumptions. Furthermore Basel II assumes constant Losses Given Default and an infinite granular portfolio which neglects the risk of random Losses Given Default and granularity. By assuming conservative values for the correlations, model risk should be absorbed as much as possible. The Basel Committee emphasizes that overall capital should not be lower under the New Accord than under the Current Accord and that the proportions of required capital between the different portfolios should not change. Therefore the asset correlation constitutes the adjustable item when the capital curves are calibrated.

Irrespective of the motivations for using correlations when capital rules are determined, any backtesting of PD forecasts by supervisors should be accomplished under more realistic assumptions about the parameters. To see the implications we provide a simple example. If a bank provides a PD forecast of 1% (for example for rating grade BB) the supervisor who tests this forecast could calculate a quantile of the default distribution, for example the 99% quantile. If the actual realization of the default rate exceeds this quantile the validity of the PD forecast is rejected.

Assuming a 20% asset correlation the 99% quantile of the distribution of next year's potential defaults in a very large portfolio is about 7.53%. That is, an observation of, say, 7% for the empirical default rate would not be enough evidence against the adequacy of a predicted PD of 1%. However, looking at empirical default rates, peaks which exceeded this quantile do not seem realistic (see for example the default rates in Exhibit 1 and the peaks therein).

Our estimations showed that the largest asset correlation is about 2.3%, even if a static model is used. If an upper benchmark of for example 3% for the asset correlation is used, the 99%

quantile is only 2.55%. The distribution of potential defaults is much more narrow (see Exhibit 6). Then a rejection of the hypothesis says that either the PD forecast is wrong, or that the true correlations in the tested portfolio are larger, or both. In either case the supervisor receives a signal of more potential risk than allowed and could decide to examine the rating system more closely. Thus, a hypothesis about PDs could be backtested by supervisors in practice if realistic values of upper benchmarks for the asset correlations are assumed.

---Insert Exhibit 6 about here---

The model can also be easily used by financial institutions for parameterizing their credit risk models. An example for the dynamic model is shown below. Regarding the CreditMetrics approach as described in Gordy (2000) the unconditional dynamic default probabilities within a segment can be obtained by

$$\lambda_t = \Phi\left(\beta_0 + \beta_1 \; DR_{t-1}\right) \qquad (*\ 10).$$

The variances of the conditional default probabilities are derived by

$$\begin{aligned} \sigma_t^2 &\equiv \mathrm{Var}\left(\lambda_t\left(f_t\right)\right) \\ &= \Phi\left(\beta_0 + \beta_1 \; DR_{t-1}, \beta_0 + \beta_1 \; DR_{t-1}, \rho\right) - \lambda_t^2 \end{aligned} \qquad (*\ 11),$$

where $\Phi\left(z_1, z_2, s\right)$ denotes the bivariate cumulative normal distribution function, or the joint default probability of two borrowers respectively. In opposite to the static models the default correlations are also dynamic and are given by[1]

[1] Whereas the *asset* correlation is assumed to be constant over time, the *default* correlation is not, see (* 12). Furthermore, not only one-year but also multi-year default correlations will change in general. However, estimating multi-year time-varying default correlations would require more observation points, i.e. longer time series and is beyond the scope of the paper. We therefore leave this field for further research.

$$\rho_t^{Def} = \frac{\mathrm{Var}\left(\lambda_t\left(f_t\right)\right)}{\lambda_t\left(1-\lambda_t\right)} \qquad (* \ 12),$$

Note that the asset correlation plays a crucial role in these expressions. To the extent that $\rho = 0$, the joint default probability becomes the product of the marginal default probabilities, thus rendering the variance in (* 11) and the default correlation in (* 12) being zero. A major advantage of this methodology is that the covariate DR_{t-1} is known when one-year forecasts for the PDs or the variances are generated. Therefore one does not need a forecast model for it which induces additional uncertainty, as it is for example necessary in the model due to Wilson (1997a, b).

In a similar way the CreditRisk+ model can be parameterized using the estimates from our model as shown in Boegelein et al. (2003). Firstly, sectors are predefined (such as countries and industries) which are analysed due to their independence using the above methodology or as described in Boegelein et al. (2002). Once the independence is ensured the parameters of our model can be transformed into the CreditRisk+ framework where the random factors are assumed to be Gamma distributed, see Koyluogly/Hickman (1998) and Gordy (2000). The unconditional default probability is given by λ_t in (* 5). Note that the parameters α_t and χ_t of the Gamma distributed conditional default probability in a given sector depend on *t* and can be obtained as

$$\alpha_t = \frac{\lambda_t^2}{\sigma_t^2} \text{ and } \chi_t = \frac{\sigma_t^2}{\lambda_t} \qquad (* \ 13)$$

where λ_t and σ_t^2 are defined as in (* 7) and (* 8).

As a simple example again sector 3 (construction) in Germany is reconsidered where the default rate in 2001 was 2.3%, see Exhibits 1 and 3 . Inserting this into the estimated function in Table 3 gives forecasts for 2002 of $\lambda_{2002} = \Phi(-2.5481 + 25.7316 \cdot 0.023) \approx \Phi(-1.96)$ $\approx 2.5\%$ (see Exhibit 3) and $\sigma_{2002}^2 = 9.96E-6$ using the asset correlation of $\rho \equiv b^2$=0.0029 from Table 3 and Exhibit 2. This leads to $\alpha_{2002} = 62.74$ and $\chi_{2002} = 0.000398$. With these parameters the default distributions under both models can be calculated as described in Koyluoglu/Hickman (1998). Exhibits 7 and 8 compare these distributions for portfolios of size 1,000 and 10,000 each. As can be seen from the figures, the differences between the CreditMetrics and the CreditRisk+ model and are indeed reconcilable. This is mainly a result of the low asset correlation, which renders both distributions almost normal since defaults are (unconditional) nearly independent. A higher number of borrowers leads to more narrow distributions. Thus, the low asset correlations firstly reduce uncertainty in the forecasts and secondly lead to an achievement of almost similar results, no matter which one of the two credit risk models is used.

---Insert Exhibits 7 and 8 about here---

Two remarks on potential future research should also be mentioned. Firstly, the model can be extended by other variables. As can easily be seen, macroeconomic figures or individual information, such as balance sheet data or ratings - where available - may be included. Applied in this way the model could constitute a bank's internal credit risk model. Furthermore, although the similarity of the outcomes for the various models is valuable and thus reduces model risk, we did not consider estimation risk which induces additional uncertainty. However, since the distribution of the parameter estimates is known and the standard errors do not exhibit dramatic magnitudes, incorporation of estimation risk should be feasible.

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Tables and Figures:

Table 1:
Segment Encoding

Segment No.	Description
1	Agriculture
2	Commerce
3	Construction
4	Manufacturing
5	Services

Table 2:
Parameter estimates of factor model for various industry sectors in the G7 countries; standard deviations are in parentheses;
estimates are significant at the 1% level, except:
* are significant at the 10% level.

		b		β_0	
Canada	1	0.07744	(0.01782)	-2.3811	(0.02515)
	2	0.07519	(0.01687)	-2.1496	(0.02403)
	3	0.04880	(0.01130)	-2.1384	(0.01576)
	4	0.05921	(0.01380)	-2.1801	(0.01922)
	5	0.06065	(0.01364)	-2.3806	(0.01937)
France	1	0.03360	(0.007975)	-2.0891	(0.01067)
	2	0.04527	(0.009687)	-1.9993	(0.01371)
	3	0.04703	(0.01009)	-1.8726	(0.01427)
	4	0.08415	(0.01789)	-1.8720	(0.02558)
	5	0.06635	(0.01412)	-2.1685	(0.02013)
Germany	1	0.07786	(0.01297)	-2.5573	(0.01753)
	2	0.08738	(0.01315)	-2.5397	(0.01890)
	3	0.1230	(0.01834)	-2.1934	(0.02675)
	4	0.09212	(0.01388)	-2.4129	(0.01995)
	5	0.09218	(0.01383)	-2.5884	(0.01996)
Great Britain	1	0.1193	(0.01995)	-3.2400	(0.02841)
	2	0.1046	(0.01646)	-2.4738	(0.02384)
	3	0.1202	(0.01883)	-2.3617	(0.02748)
	4	0.1119	(0.01755)	-1.9984	(0.02539)
	5	0.1040	(0.01631)	-2.3617	(0.02364)
Italy	1	0.06643*	(0.03697)	-2.6921	(0.02196)
	4	0.06430	(0.01265)	-2.3951	(0.01800)
	5	0.04809	(0.009463)	-2.7111	(0.01343)
Japan	2	0.06678	(0.01496)	-2.9263	(0.02137)
	3	0.1309	(0.02886)	-2.5383	(0.04256)
	4	0.1404	(0.03089)	-2.7238	(0.04604)
	5	0.05940	(0.01341)	-3.0216	(0.01902)
USA	1	0.1508	(0.02615)	-1.9329	(0.03855)
	2	0.06561	(0.01157)	-2.2482	(0.01651)
	3	0.07291	(0.01286)	-2.1817	(0.01837)
	4	0.09928	(0.01744)	-2.1889	(0.02515)
	5	0.06526	(0.01150)	-2.3080	(0.01642)

Table 3:

Parameter estimates of factor model with lagged default rates for various industry sectors in the G7 countries; standard deviations are in parentheses;

estimates are significant at the 1% level, except:
* are significant at the 10% level,
** are significant at the 5% level,
† are not significant at the 10% level

		b		β_0		β_1	
	1	0.05276	(0.01337)	-2.6549	(0.08858)	29.3112**	(9.7395)
	2	0.04375	(0.01053)	-2.6705	(0.1168)	31.2801	(7.0569)
Canada	3	0.04591	(0.01125)	-2.3244	(0.1428)	11.4035†	(8.5579)
	4	0.03841	(0.009990)	-2.5663	(0.1064)	25.3610	(7.0231)
	5	0.04621	(0.01104)	-2.7401	(0.1264)	39.9327**	(14.0490)
	1	0.02104	(0.005979)	-2.5187	(0.1139)	23.0088	(6.0810)
	2	0.02832	(0.006425)	-2.2787	(0.09753)	12.3614**	(4.1917)
France	3	0.03018	(0.006889)	-2.1475	(0.1007)	9.0964**	(3.2320)
	4	0.05521	(0.01243)	-2.1887	(0.09899)	10.3319	(3.1056)
	5	0.04019	(0.009027)	-2.4673	(0.08755)	19.8487	(5.6124)
	1	0.04079	(0.008569)	-2.8252	(0.05016)	52.4558	(9.4293)
	2	0.03866	(0.006123)	-2.8310	(0.03769)	53.6563	(6.7212)
Germany	3	0.05341	(0.008389)	-2.5481	(0.04265)	25.7316	(2.9868)
	4	0.04611	(0.007311)	-2.7188	(0.04393)	39.4392	(5.4980)
	5	0.04319	(0.006757)	-2.9236	(0.04224)	71.7143	(8.8346)
	1	0.06794	(0.01301)	-3.4266	(0.04298)	308.04	(64.5042)
Great	2	0.06357	(0.01040)	-2.7018	(0.05534)	34.6611	(7.9420)
Britain	3	0.06036	(0.009936)	-2.6344	(0.04363)	29.9497	(4.5336)
	4	0.07201	(0.01173)	-2.3055	(0.06592)	13.3229	(2.7515)
	5	0.06173	(0.01003)	-2.6241	(0.04909)	29.0017	(5.1902)
	1	0.03588**	(0.01296)	-2.7536	(0.09419)	15.2097†	(26.6097)
Italy	4	0.03994	(0.008296)	-2.6998	(0.06621)	36.4961	(7.8921)
	5	0.03282	(0.006771)	-2.9582	(0.06293)	72.8958	(18.4089)
	2	0.02804	(0.006767)	-2.9553	(0.05125)	25.5201	(29.2029)
Japan	3	0.03998	(0.009582)	-2.7656	(0.04102)	44.5194	(7.1703)
	4	0.04283	(0.01031)	-2.9409	(0.04291)	73.7335	(12.9681)
	5	0.02207	(0.005507)	-3.0711	(0.04365)	50.7087†	(34.4750)
	1	0.1283	(0.02316)	-2.1815	(0.09912)	8.6034**	(3.4154)
	2	0.05360	(0.009786)	-2.5184	(0.09114)	21.2711**	(7.1735)
USA	3	0.05809	(0.01062)	-2.4176	(0.07817)	16.0312	(5.1900)
	4	0.06596	(0.01208)	-2.5689	(0.08348)	25.0736	(5.5321)
	5	0.05268	(0.009610)	-2.4504	(0.07897)	14.1716*	(7.3956)

**Exhibit 1:
Default rates of segment 3 (construction) in various countries**

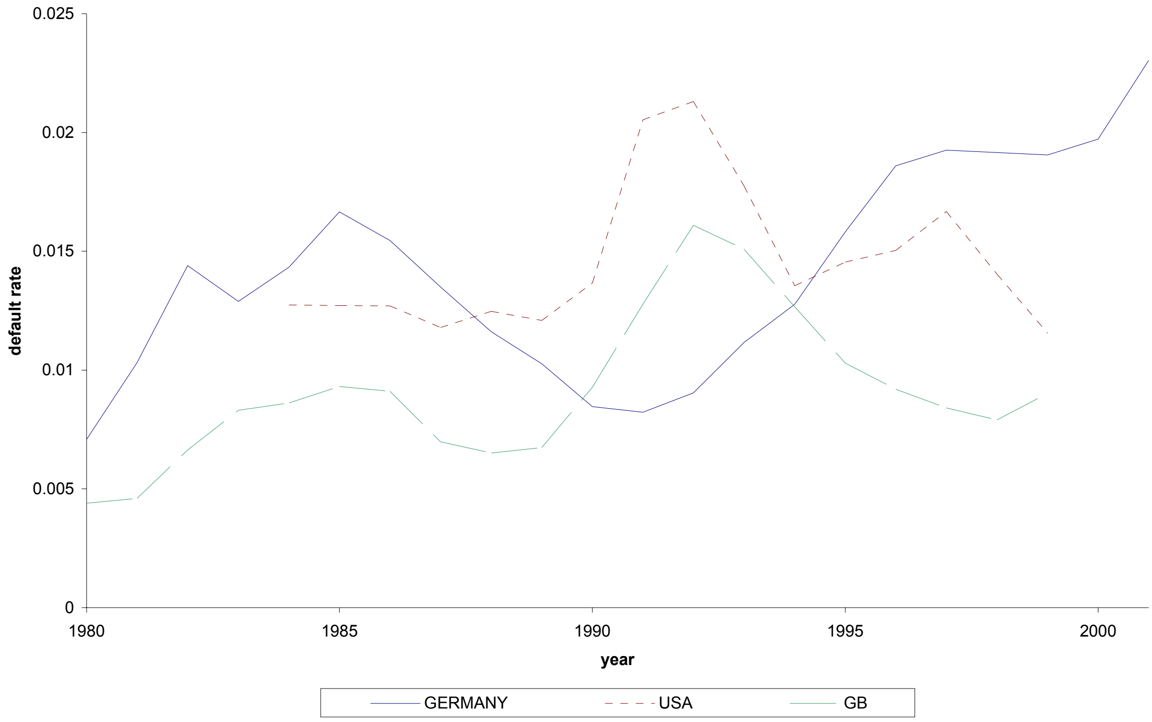
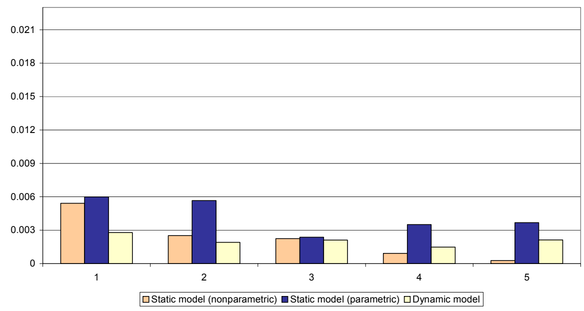
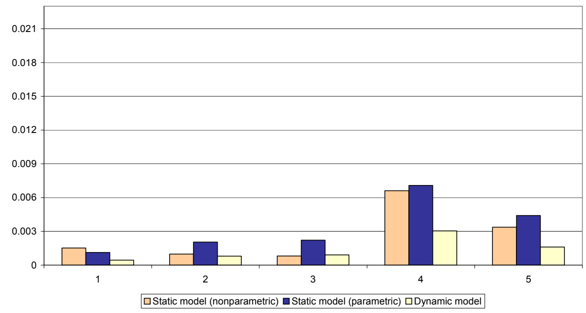


Exhibit 2:
Asset correlations by country and industry for various models

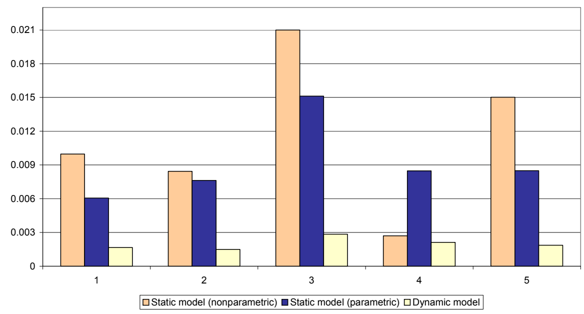
Canada



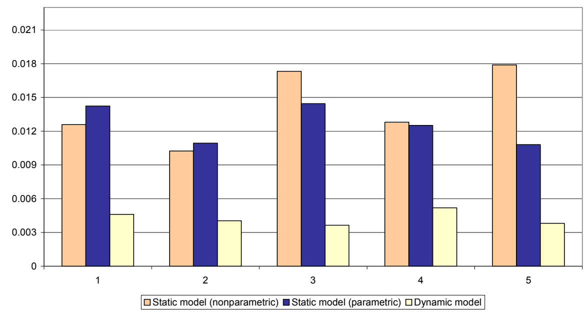
France



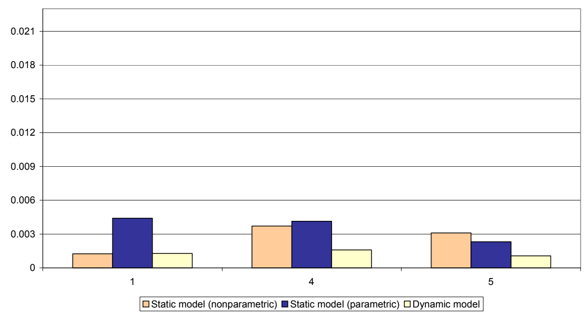
Germany



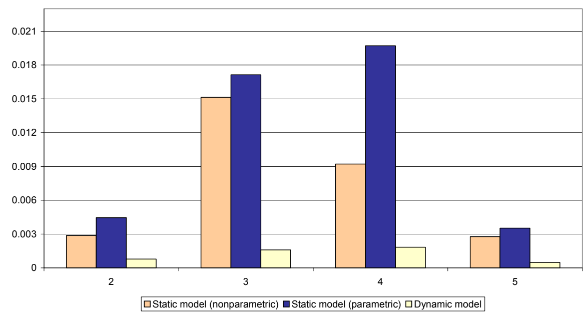
Great Britain



Italy



Japan



USA

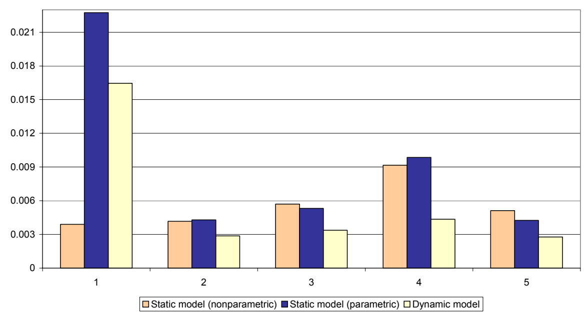


Exhibit 3:
Actual default rates in the construction sector in Germany versus fitted values/forecasts and average default rate

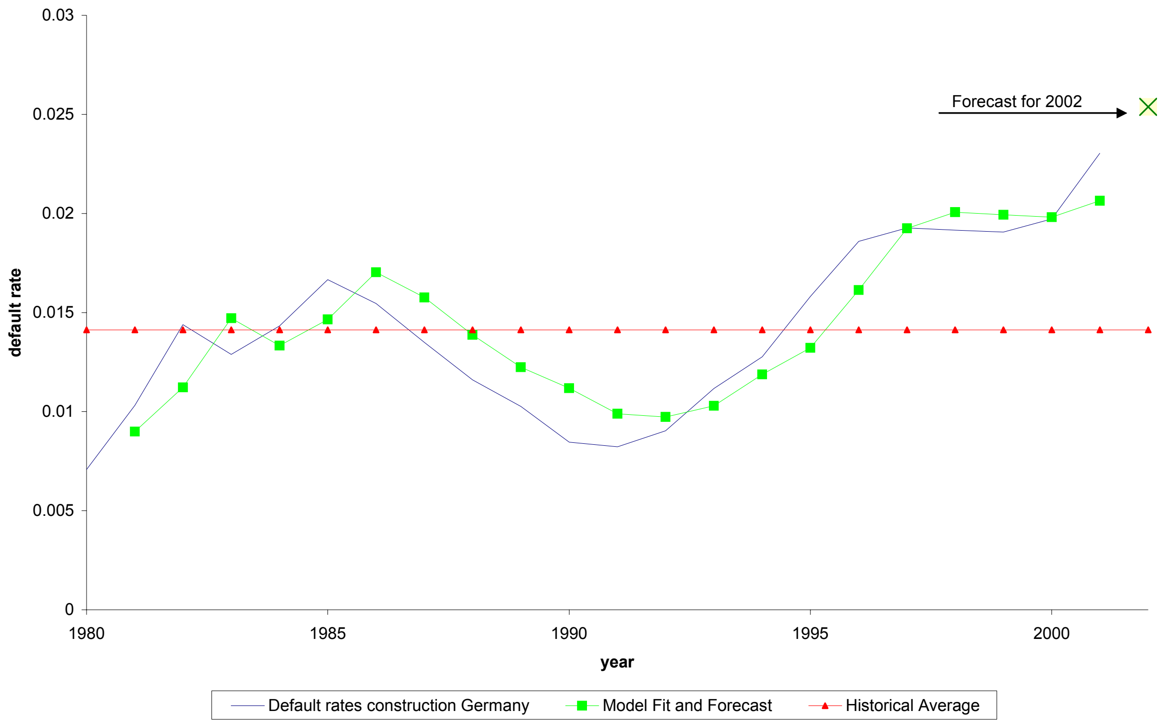


Exhibit 4:
Estimated asset correlations for borrowers from different countries; static model

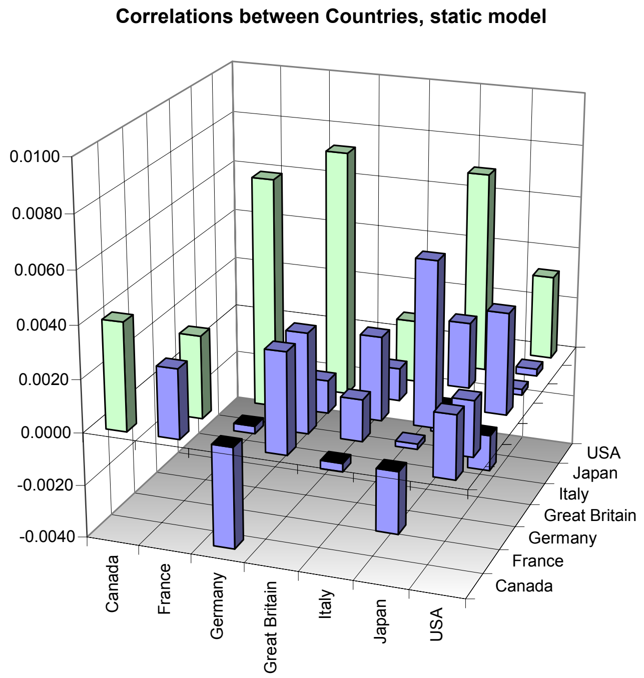


Exhibit 5:
Estimated asset correlations for borrowers from different countries; dynamic model

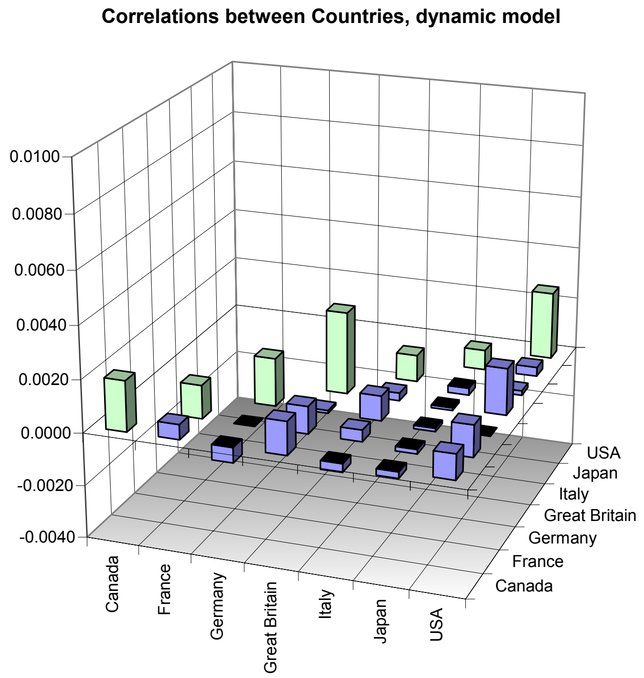


Exhibit 6:
Distributions of potential one-year defaults under Asset Correlations of 20% and 3%; Portfolios are very large; each PD is 1%

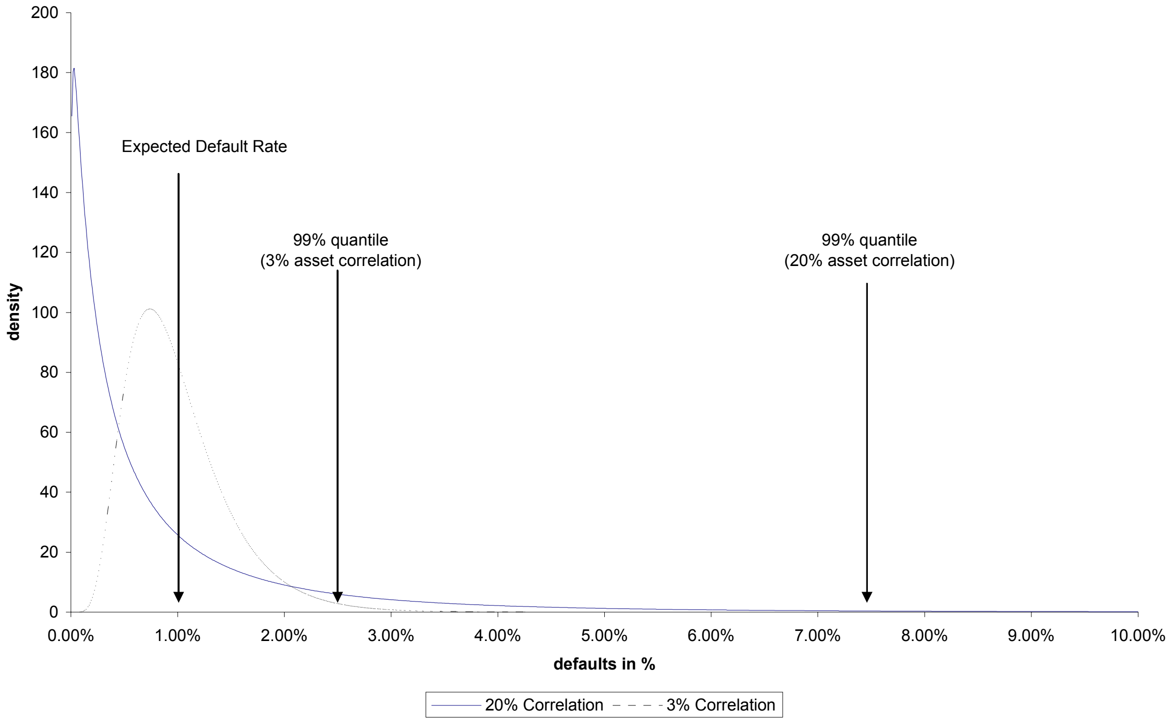


Exhibit 7:
Distributions of potential one-year defaults for construction sector, Germany, using CreditRisk+ and CreditMetrics model, N=1,000 borrowers

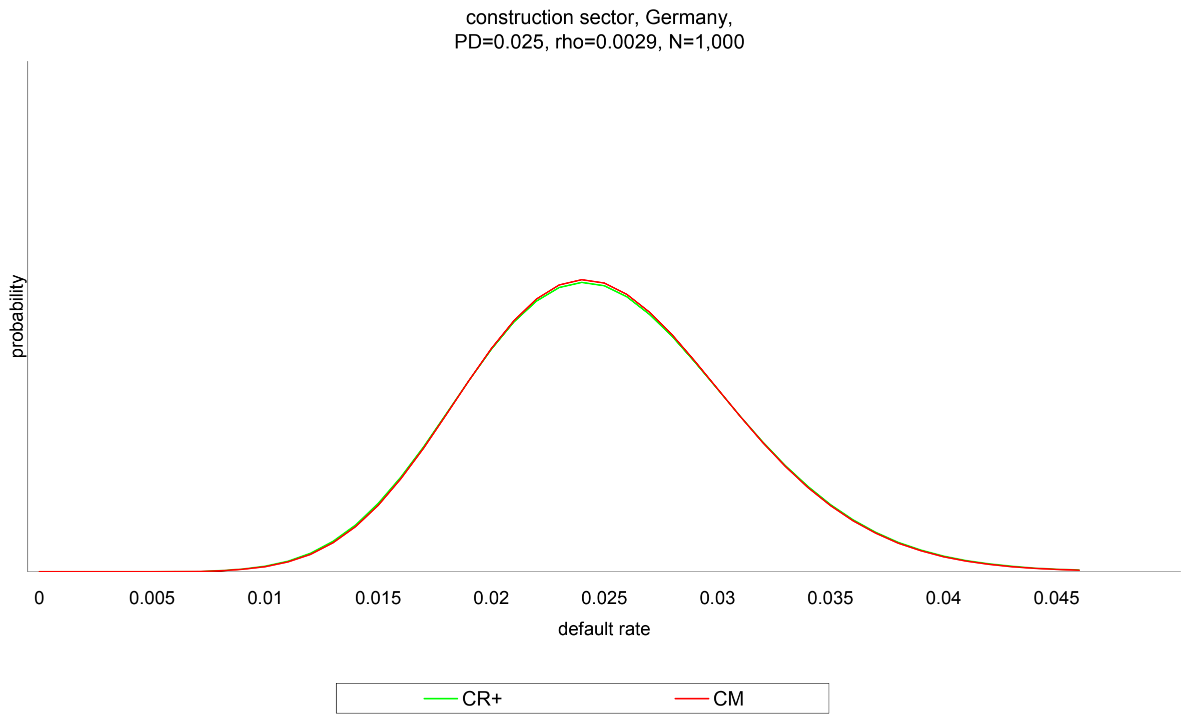


Exhibit 8:
Distributions of potential one-year defaults for construction sector, Germany, using CreditRisk+ and CreditMetrics model, N=10,000 borrowers